

Abstract

In this paper, I discuss about the "Teaching Experiment Methodology" following the article of Steffe & Thompson: Teaching Experiment Methodology- Underlying Principles and Essential Elements. Here I briefly discuss what a teaching experiment is, what are its essential components, what are some of the reasons for its emergence as well as some reasons for its acceptance as a research tool. I portray how teaching experiments can be used in the classroom for mathematics instruction.

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INTRODUCTION

Many of you are wondering why I always choose papers for annotations related to constructivism or Steffe's research works. Last summer I took EMAT 8030 with Dr. Steffe. Being a fifth year mathematics Ph.D. student, I am used to playing with abstract concepts in mathematics. But I should confess here, EMAT 8030 class challenged me to think in a non-conventional way by introducing me to the idea of "Constructivism". Though summer session was too short to delve deeper into the area, it was enough to raise my curiosity to learn more about it. As a mathematician, I cannot just let it go without completely understanding the ideas and principles and related work on constructivism. EMAT 7050 class gave me an opportunity not only to read papers for the annotated bibliography but also to present my own understanding from the readings in the form of this paper.

For this paper, I chose to discuss Teaching Experiment Methodology. Teaching experiment is a kind of research in mathematics education that emerged in the 70's in the United States. The Teaching Experiment Methodology was widely used in Soviet studies in mathematics education prior to the 1960-1970. Many of these studies were translated into English and published by a joint project of the University of Chicago (Wirzsup), Stanford University, and the University of Georgia. Through teaching experiments, researchers try to understand the mathematical concepts and operations of students. Steffe has mentioned, "a primary purpose for using teaching experiment methodology is for researchers to experience, firsthand, students' mathematical learning and reasoning" (Steffe & Thompson, 2000, p. 267).

RATIONALE

In this paper, I propose to discuss what a teaching experiment is, what are its essential components, what are some of the reasons for its emergence as well as some reasons for its acceptance. Next, I propose to discuss how teaching experiments can be used for mathematics instruction

CONSTRUCTING KNOWLEDGE

When we perceive familiar things, it appears to us that we know them as they fit into our previously developed understanding. Piaget referred to this mental action as *assimilation* i.e. fitting our experiences or perceptions into existing ideas (Van de Walle, 1994). Other times, we cannot fit current ideas into previously developed understanding and that creates disequilibrium. To maintain equilibrium our ideas must be modified or new ideas must be created to fit the current idea. This action is called accommodation. The theory of constructivism is based on the mental activities of assimilation and accommodation. "Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the (knowing) subject" (von Glasersfeld, 1990, p.22).

This way of constructing knowledge is not seen in the current school system. The current school system considers mathematics in the school textbooks as what children should learn. This belief places mathematics outside the minds of the children who are yet to learn it. In constructivist view, children need to make connections between old ideas and new ones to construct and understand new ideas. As Steffe said (Steffe & Olive, 2010):

Because children's mathematical learning in school occurs in the specific context of teaching, it might seem to be reasonable to regard the content of children's mathematical knowledge to be explained by conventional mathematical concepts. However, several researchers working within a constructivist view of knowledge

and reality have found it necessary to explain what students learn using constructs that differ significantly from standard mathematical concepts and operations. (p. 14)

That being said, it is important to understand how students are thinking and what are their ways and means of operating.

FIRST ORDER AND SECOND ORDER MATHEMATICAL KNOWLEDGE

According to Steffe, first order mathematical knowledge is one's own mathematical knowledge. "It is the models an individual construct to organize, comprehend, and control his or her experience" (Steffe & Olive, 2010, p. 16). The second order mathematical knowledge is "the models observers' construct of the observed person's knowledge" (p. 16). Observers use their first order mathematical knowledge to construct the second order mathematical knowledge of the observed person. Steffe called the second order models as social models as they are constructed through social processes (Steffe & Olive, 2010). It is critical to distinguish between first- and second- order mathematical knowledge to avoid mixing between children's mathematical concepts and operations with conventional school mathematics (Steffe & Olive, 2010).

STUDENTS' MATHEMATICS, MATHEMATICS OF STUDENTS AND MATHEMATICS FOR STUDENTS

Students' mathematics constitutes students' mathematical realities. It is indicated by what students do as they engage themselves in mathematical activities. "Children's mathematics constitutes children's first-order mathematical knowledge" (Steffe & Olive, 2010, p. 16). Students' mathematics is independent of the observer's mathematics.

Mathematics of students refers to models of students' mathematics. It also includes the various modifications students make in their ways of operating. The goal of a researcher in a teaching experiment is to construct mathematics of students. Steffe said mathematics of students is considered as a legitimate mathematics as long as researchers can find rational grounds for what students say and do (Steffe & Olive, 2010). "Looking behind what students say and do in an attempt to understand their mathematical realities is an essential part of a teaching experiment" (Steffe & Thompson, 2000, p. 269). The process of analyzing what students say and do is called conceptual analysis.

Mathematics **for** students consists of the concepts and operations that students might learn (Steffe & Olive, 2010). Mathematics for students is the mathematics of some other students whose second-order models are created by the observers. Steffe does not regard those concepts being part of the observers' mathematical knowledge but they are the mathematics students learn as observed by the observer (Steffe & Olive, 2010). Teachers can know mathematics for students by observing students' mathematical activity. Teachers are responsible to understand mathematics **of** students and mathematics **for** students. As Steffe (2010) said:

Mathematics for children can be known only through interpreting changes in children's mathematical activity. Specifically, the mathematics for a group of children is initially determined by the modifications that other children have been observed to make whose mathematical behavior is like the current children. (p. 17).

REASONS BEHIND THE EMERGENCE OF TEACHING EXPERIMENTS

Teaching experiments in mathematics education emerged in the Unites States after 1970.

The reasons for the emergence of teaching experiments are as follows:

- Models outside of mathematics education- Models for mathematics of students were developed outside of mathematics education and for the purposes other than educating students. Models were needed that included students' progress as a result of mathematical interaction. There was a need for researchers to learn to use their own mathematical knowledge rather than using known mathematical systems in actual interactions with students.
- Gap between the practice of research and of teaching- There was a big gap between the practice of research and the practice of teaching. Prior to teaching experiments, researchers used to select one or more samples from a target population and subjected them to various treatments. Then the effects of the treatments were compared with the intention of specifying differences between or among them. These experimental designs suppressed conceptual analysis. "The subjects in the experiments were recipients of treatments and usually not the focus of conceptual analysis. The subjects were subjected to treatments; they did not participate in the co-construction of the treatments in the context of teaching episodes. How students made meanings or the meanings they made was not of primary interest" (Steffe & Thompson, 2000, p. 271).

REASONS FOR ACCEPTANCE OF TEACHING EXPERIMENTS

According to Steffe and Thompson, there were several reasons for the teaching experiments to be accepted by the mathematics educators.

• The methodology seemed to be intuitively correct. "The word "teaching" in the title appealed to the common sense of mathematics educators and resonated with

- their professional identification as mathematics teachers" (Steffe & Olive, 2000, pp. 271-272).
- Versions of the methodology were already being used by the researchers in the
 Academy of Pedagogical Sciences in then Union of Soviet Socialist Republics.

 This provided a sense of respectability in the mathematics education research
 community.
- There was a shift in researchers to understand students' mathematics. Researchers started acknowledging that mathematical activity in school occurs as a result of student's participation in teaching. Experimental methodologies used in 1970s were not able to address these issues. Teaching experiment methodologies emerged to support this new wave of thinking. Teaching experiment methodology is a conceptual tool that researchers use in the organization of their activities.

DIFFERENCE BETWEEN CLINICAL INTERVIEW AND TEACHING EXPERIMENT

The clinical interview is aimed at understanding students' current knowledge and the teaching experiment is directed toward understanding the progress students make over an extended period of time. Teaching experiment is a living methodology designed to explore and explain students' mathematical activities. This methodology is not prescriptive, it is contextual.

THE ELEMENTS OF TEACHING EXPERIMENT METHODOLOGY

A teaching experiment consists of a sequence of teaching episodes. The following are the four basic elements of teaching experiment methodology: A teaching episode includes a teaching agent, one or more students, a witness of the teaching episodes and a method of

recording the entire teaching episode. These records can be used in preparing subsequent episodes as well as in conducting a retrospective conceptual analysis of the teaching experiment.

The first important key element in this methodology is exploratory teaching. Researchers in Steffe et al. (1976) realized that teaching students for short period of time could not form a solid understanding of students' thinking. It is important to get thoroughly acquainted with students' ways and means of operating in the respective area of interest of the researcher. It is really important for teacher-researcher to put aside his/her own concepts and operations. The teacher-researcher should refrain from insisting students to learn what he/she knows. As Steffe said, "the researcher's mathematical concepts and operations can be orienting, but they should not be regarded, initially at least, as constituting what the students should learn" (p. 274). The main purpose for engaging in exploratory teaching is to make essential distinctions between students' ways and means of operating.

The second element in the teaching experiment is testing research hypotheses. Teaching experiments are done to test hypotheses as well as to generate them. Formulation of the research hypotheses prior to a teaching experiment guides the initial selection of the students and the general intentions of the researcher. In addition to formulating and testing major research hypotheses, researchers need to generate and test hypotheses during the teaching episodes, in between teaching episodes. The teacher-researcher might formulate one or more hypotheses to be tested in the next episode. The main job of the teacher-researcher is to continuously postulate meanings that lie behind students' actions. This is the way students guide the teacher-researcher. Due to unanticipated ways and

means of operating, the teacher-researcher may be forced to abandon his hypotheses and create new hypotheses and situations on the spot. The basic challenge for the teacher-researcher is "rather than believing that a student is absolutely wrong or that the student's knowledge is immature or irrational, the teacher-researcher must attempt to understand what the student can do; that is, the teacher-researcher must construct a frame of reference in which what the student can do seems rational" (p. 277).

The next component is teaching. Teaching in a teaching experiment occurs while interacting with students. Researchers might not know where they are headed but they are obliged to engage in responsive and intuitive interactions with the students. The goal of the researchers is to explore students' reasoning by interacting with students in a responsive and intuitive way. At this point, the teacher-researcher has no expectation of what the children can do. The teacher-researcher becomes more experienced with the students as the teaching experiment progresses and can interact with students more analytically. Analytic interaction is "an interaction with students initiated for the purpose of comparing their actions in specific contexts with actions consonant with the hypothesis" (pp.280-281). The goal of a teacher in a teaching experiment is to build living models of students' mathematics. This is possible only when students are selforganizing and self-regulating. If students are not self-organizing and self-regulating, they would not modify their previous schemes. In a teaching experiment, the contributions from the students have to be independent of the teacher-researcher in a way that teacher-researcher is responsible for creating situations where students can use their mathematical schemes independently and teacher-researcher can observe independent modifications in their use of schemes.

Next is the role of a witness in a teaching experiment. The presence of a witness-researcher in a teaching experiment is a key element of the teaching experiment. Sometimes the teacher-researcher might get caught up in trying to respond to what the student just said or did and would not reflect on the student's contribution. In this situation, a witness-researcher can help the teacher-researcher both to understand the student and to posit further action. Sometimes the witness-researcher can catch important elements of a student's actions that might have missed by the teacher-researcher. It is important for both the teacher-researcher and the witness-researcher to be part of the next teaching episodes.

The next element is retrospective analysis and model building. Retrospective analysis is an important component of teaching experiment. Carefully analyzing the video records, the researcher has the opportunity to activate the records of their past experiences with the students and bring into conscious awareness. These give insight to students' actions and interactions that were not available to the teacher-researcher during the teaching interaction. In the model building process, the following concepts are used: assimilation, accommodation, cognitive and mathematical play, mental operations, schemes, etc. "These concepts emerge in the form of specific and concrete explanations of students' mathematical activity" (Steffe & Thompson, 2000, p. 293). Since the models are based on the interaction with students, it is expected that these models will be useful in further interactive mathematical communication with other students.

ARE TEACHING EXPERIMENTS SCIENTIFIC?

Teaching experiments are concerned with conceptual structures and models of the kinds of change that are considered learning or development. It is to be noted that no single

observation can indicate learning or development. At least two observations made at different times are required to understand change in learning. The fundamental point in a teaching experiment is that the researchers are trying to learn what change they can bring forth in their students and how to explain such change. "Regardless of whether students change as anticipated or change in ways that are known only in retrospect, the researchers do become aware of a directionality of change" (p. 295), which is considered unusual in other kinds of research. Often preliminary observations are reinterpreted on the basis of the analysis of the videotapes, which might be deemed improper in other branches of science. But "it is no less legitimate than, for example, the reevaluation of microbiological evidence on the basis of enlargements of a microscopic image" (p. 295). In the teaching experiment, formulation of hypothesis, testing and reconstruction of the hypothesis form a recursive cycle. Steffe found that students' mathematical schemes change slowly over time and students work at the same learning level for extended periods. If the researchers find the inferences made about students' mathematics compatible with observations, it does not mean that the researchers have insight into workings of the students' minds. The construction of the mathematics of the students is based on a conceptual analysis of the mathematical language and actions of students as well as on theoretical constructs perhaps established in an earlier conceptual analysis not involving students. The construction of models that can be seen in some way as analogous to a student's thought processes is the main scientific purpose of teaching experiments. A teaching experiment includes the generation and testing of hypotheses to see whether or not the students' action can be interpreted by the model. Hence, teaching experiments are scientific.

ROLE OF TEACHING EXPERIMENT IN TEACHING

Teaching experiments can be used to construct mathematical schemes. Schemes are the models of one's way of operating. Mathematical schemes refer to students' mathematical language and actions. Students' ways of operating are deeply connected to how they make meaning out of their mathematics. According to von Glasersfeld (1980), schemes consist of three components: Trigger or occasion; mental actions or operations triggered by the occasion; and the result or sequel of the activity (Fig. 1). The first component, trigger or occasion, is an experiential situation (Steffe & Olive, 2010). This component of a scheme is a situation, which is related to a task that a child perceives. The second component is the child's activity in response to the situation and the third component is the result of the child's activity.



Figure 1: Three components of an operational scheme.

Schemes can be developed using the teaching experiments. These schemes can be used by teachers and researchers to model students' current cognitive constructs. Schemes can be used further to predict students' actions (Norton & McCloskey, 2008). Though it is common to misinterpret schemes as strategies. Norton & McCloskey (2008) demonstrated how strategies and schemes differ which is shown in the table 1.

Table 1: Difference between Strategies and Schemes

STRATEGIES	SCHEMES
Strategy is a way of operating that usually occurs inside student's awareness.	Scheme is a way of operating that usually occurs outside student's awareness.
Strategies are activated in a procedural manner.	Schemes are activated at once.
Strategies are students' construct to solve a given problem.	Schemes are teacher constructs to understand students' mental actions and for predicting students' action.

If teachers understand students' current schemes, teachers can support students' actions by providing various tasks which can frustrate students' ways of operation so that students can think of new ways of operations. Every year, teachers learn about the Math Recovery Program, "which uses Steffe's whole- number schemes to model, understand, and support children's development of whole-number reasoning" (Norton & McCloskey, 2008, p. 48).

CONCLUSION

In this paper, I gave an overview of the teaching experiment methodology. I have also shown how teaching experiments can be used in teaching. The models from the teaching experiments can be used by teachers to understand their students' actions and ways of operating.

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